A MATHEMATICAL MODEL OF CARRIER AIRCRAFT OPERATIONS IN ANTI-AIR WARFARE

F. I. COLLINS, JR.

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Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

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ABSTRACT

This paper presents a mathematical model of carrier aircraft operations in anti-air warfare. The model developed herein was a necessary outgrowth of a large scale study conducted by Stanford Research Institute (SRI) for the Navy. One of the problems facing SRI was determining the number of aircraft stations that a carrier task force could fill continuously over some given time period. Therefore the measure of effectiveness produced by the model is a probability distribution over the number of Combat Air Patrol (CAP) or Airborne Early Warning (AEW) aircraft able to be maintained on station as a function of time in an operating area.

The model is developed as a time dependent Markov process with an appropriate stochastic matrix of transition probabilities. The maintenance process is described mathematically as a pulsed input, multiple server queue. The model has been programmed for use on a high speed digital computer, the CDC-1604, installed at the U. S. Naval Postgraduate School, with arbitrarily specified parameter values. The computer version of the model with operating instructions is currently available at the school.

PREFACE

This paper was started as a summer project by the author at the Naval Warfare Research Center, Stanford Research Institute, Menlo Park, California in June, 1960. The basic ideas and concepts were formulated jointly by Mr. Jack A. Butler, Dr. Donald Guthrie Jr., both of SRI, and the author. Further development and translation of the model into a computer program was done by the author at the U. S. Naval Postgraduate School, Monterey, California from July 1960 to March 1961. Although the original concept did not include a sensitivity analysis of parameter values, the author believes that the development would be incomplete without a discussion of input parameters and the effects of varying their values.

A problem similar to the one considered herein has been studied by the RAND Corporation, using an Air Force squadron of aircraft with a complex maintenance system. The RAND model, reported in RM-2374, is much more detailed in maintenance aspects, but it is a Monte Carlo model. Consequently a large number of runs with the same set of parameter values are required to place confidence in the results. It was our belief that the maintenance simulation could be simplified and the entire operating cycle be abstracted as a Markov process. In this respect we were successful, but there remain questions to be answered.

Perhaps the most important is "how closely does the model approximate the system it describes?" There is a lack of satisfactory statistical data to answer this question, for several reasons:

1. It is believed that the concept of maintenance "spots" as developed in assumption 10, page 6, is a novel but useful one;

2. The last full scale operation under combat conditions for which any information is available is the Korean War;

3. The maintenance reporting procedures currently in use by the Navy do not provide the information required for inputs to this study.

Attempting to answer the above question by collecting and analyzing data is considered a (large) separate problem. However, the answers obtained from over 500 runs of the computer model with greatly varied inputs always produced answers that were credible. If the collection of data from large scale fleet exercises could be undertaken with this model in mind, the results could be compared with operational experience and an indication of "goodness of fit" would be obtained. Further, it might be possible to refine the model to more closely approximate the real world system, and provide a dynamic model that could be used by operations planners for a wide range of conceivable situations. Even in its present state of development, the model seems satisfactory for providing quantitative answers to a problem that has previously been solved only by estimates of the planner. It is believed that the values of "upper and lower bounds" on the answers discussed in the sensitivity analysis is particularly valuable to the Navy.

I wish to express my sincere gratitude to Professor Franklin F. Sheehan, who in class introduced me to many of the mathematical concepts used in this paper, and as faculty advisor provided the encouragement and assistance necessary to apply these concepts; to Professor Jack A. Borsting for his helpful advice as second reader; to Dr. Donald Guthrie Jr. and Mr. Jack A. Butler of SRI, whose names

should appear as co-authors of several ideas in this paper; and to my wife, who, while enduring many lonely hours of computation and writing, provided me with the inspiration and clerical assistance necessary to do this work.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS

Symbol	Page first mentioned	Definition or meaning
a/c	viii	aircraft
λ	6	mean repair rate of aircraft
t	11	time
E_k	11	outcomes or states
P	11	matrix of transition probabilities p_{ij}
p_{ij}	11	conditional probability of observation at $t+1$ producing outcome E_j, given observed outcome at t was E_i
$Q^{(t_o)}$	11	an initial probability distribution over the states E_k
$q_k^{(to)}$	11	the probability of a system being in state E_k initially
t_o	11	time of start of a/c operations in the operating area
$X_a(t)$	12	the number of a/c flying at time t, not having flown in previous launch-to-launch interval
$X_b(t)$	12	the number of a/c in or awaiting maintenance at time t
N	12	total number of a given type a/c aboard a carrier
A	12	desired number of a/c to be maintained on station
T	13	time after t when a/c from previous launch are recovered
e	13	short length of time
$X(t)$	14	vector $= \left[X_a(t),\ X_b(t)\right]$
$Q^{(t)}$	14	a probability distribution over the states E_k at time t, or the probability that $X(t) = (\alpha, i)$

Symbol	Page first mentioned	Definition or meaning
Υ_{fgh}	27	the probability that of f a/c available for launching, g will be launched and h will enter maintenance
$s = 1 - p_{\Upsilon}$	27	the probability that an a/c will be successfully launched
p_{Υ}	27	the probability of an a/c having equipment failure just before, during, or just after launch (i.e., the probability of a "deck dud")
p	28	the probability of an a/c having equipment failure during flight
$\Pi_{\alpha}(m)$	28	the probability that of α a/c flying, m will enter maintenance upon return to the carrier
D	28	the number of maintenance "spots" available aboard a carrier
$P_{ij}(\Upsilon)$	28	the probability that the maintenance queue changes in length from i to j a/c in time Υ

CHAPTER I

THE PROBLEM

The broad problem considered in this study is typical of those facing any operations planner. It may be stated as follows: "given a certain number and type of aircraft, and a certain desired coverage of a defensive area, what is the optimum stationing policy for these aircraft? How many shall be on station at all times, and where should these stations be? If these values are fixed, what are the chances that the coverage will be satisfactory for the period under consideration? What are the effects of changing any specific stationing plan? Finally, what confidence may be placed in the answers obtained?" The planner or decision maker considers all information or data that he considers pertinent to this problem, and derives answers to the above questions. The answers are a function of his experience, estimates, and assumptions. If he concludes that it is unlikely the desired coverage will be maintained with the given number of aircraft, he may recommend augmenting this number to improve the chances of success.

The model developed in the following pages will provide quantitative answers to the above questions. Its development is in the logical sequence of:

1. Consideration of the initial conditions;
2. Assumptions;
3. Inputs derived from the initial conditions and assumptions;
4. Formulation of the model to translate inputs into outputs;
5. Sensitivity of the model to changes in the initial conditions and assumptions.

Although the model was designed as representing aircraft carrier operations, it may be readily applied to any type aircraft squadron with similar operating procedures.

CHAPTER II

THE MODEL -- INITIAL CONDITIONS

The initial conditions presented are those required to characterize a typical carrier task force operating at sea either at present or some future time. They essentially limit the study to answering specific questions as mentioned in the statement of the problem. Conclusions based on the answers obtained are not drawn for any related questions, e.g., are more carriers needed in a task force requiring a specified defensive coverage?, etc.

The carrier task force is composed of two CVA-59 class attack carriers and other surface units. A certain percentage of the total deck space available on each carrier will be alloted to defensive aircraft. Both Airborne Early Warning (AEW) and interceptor type aircraft (CAP) will be loaded to conform to this percentage allocation. Standard CVA operating procedures will be utilized for flight operations, with recovery shortly following launch of succeeding flights to reduce time into the wind as much as is practicable. Flight scheduling will reflect the above policy also, consistent with endurance times of specific aircraft types.

A defensive zone will be established for the task force. In this zone, continuous surveillance by AEW aircraft and continuous coverage by interceptors is desired. The required aircraft may be provided by both carriers jointly, or by alternating "duty carriers". The AEW aircraft will be one specific type, as will the interceptors. However, consideration will be given to using different types of interceptors for the entire operation, in order to compare aircraft with different input

parameters.

Task force disposition will not be considered other than fixing the general shape of the defensive area in order to determine approximate values for the number of aircraft stations to be filled. Similarly, surface unit equipment configuration will not affect the stationing of aircraft and no attempt will be made to match surface defense capability with that of CAP, since this study is concerned only with the air aspects of defense.

CHAPTER III

THE MODEL – ASSUMPTIONS

1. In order to assure full CAP and AEW coverage at all times during the cycle, aircraft will be relieved on station.
2. The percentage of the total CVA-59 deck space allocated to anti-air warfare aircraft will be held constant. The quantities of interceptor and AEW aircraft will be varied within this fixed deck space, but the percentage will not vary. The attack aircraft capability will therefore be the same regardless of what type interceptors are aboard.
3. It is assumed that the duration of the CAP and AEW cycles will be 24-72 continuous hours. The model has been programmed for 72 hours to illustrate the outside limit.
4. The maintenance crews will have sufficient notice prior to the beginning of the CAP cycle to complete the 60 and 120 hour checks on all interceptor and AEW aircraft. It will not therefore be necessary to perform these checks during the CAP cycle.
5. It is assumed that no aircraft parts will be delivered to the carriers during the CAP cycle. This assumption is realistic for the following reasons:
   a. The task group will be within range of enemy aircraft during the CAP cycle which would make delivery very precarious.
   b. Delivery aircraft might assist the enemy in pin-pointing the task group's location.
   c. The task group will be far removed from supply points, which would make it improbable that parts shortages that develop could

be remedied during the 72 hours.

6. Once an aircraft becomes "Aircraft Out of Commission Parts" (AOCP) it will remain in that status for the balance of the CAP cycle (see Assumption #5).

7. Every interceptor on a CAP station will always have its full combat fuel reserve so that it can be recruited to intercept raiders if necessary. The CAP aircraft will be relieved in sufficient time so that they will never need to use the reserve except in combat.

8. Only minor maintenance will be performed on the flight deck during the duration of the CAP cycle.

9. Except in extreme emergencies, aircraft will be recovered by the carrier from which they were launched. Since carriers will be loaded with a full complement of aircraft, the recovery of aircraft from another carrier would cause severe space problems.

10. Maintenance for all aircraft of any given type will be conducted as follows;

   a. Crews will be formed into a number of groups, and will be assigned certain areas on the hangar deck in which maintenance may be conducted.

   b. Aircraft will be repaired in these maintenance "spots" by these crew teams. The teams may be composed of similar or different rating groups.

   c. All types of maintenance may be grouped as one class, with no distinction among airframes, engines, electronics, or other types.

   d. Each of the maintenance "spots" will repair aircraft at some mean repair rate, say λ, and this mean repair rate is identical for each spot.

e. The maintenance crews will work in these spots for the entire duration of the planned 72 hour cycle, probably on a watch basis.

11. The probability that an aircraft will have a failure requiring maintenance immediately prior to, during, or just after launching (i.e., become a "dud" at launch) is identical for similar type aircraft, but independent within that type.

12. The probability that an aircraft will have a failure during flight that will require maintenance upon return is also identical for the same type and independent within type. Both assumptions 11 and 12 say that the failure of any specific aircraft will not affect the probability that the remainder of its type will fail when launched or in flight.

13. No aircraft will be lost due to enemy action or casualties aboard ship. If attrition is expected, the expected number of aircraft lost during the operation may be deleted from the initial loading and the resulting answers should represent lower bounds on solutions.

14. Aircraft becoming AOCP during the operation will be deleted from the initial loading before commencement of flight operations, again giving a lower bound. Assumptions 13 and 14 are made to keep the number of states under consideration at any given time within some reasonable size. If this is not done, the capacity of the computer memory for which the mathematical model is programmed will be greatly exceeded, and time required for computation would rise by at least a factor of four.

CHAPTER IV

THE MODEL -- INPUTS (OR PARAMETER VALUES)

The inputs used in the model were derived from the initial conditions and assumptions. For any simulation of real world events, the inputs must be realistic and reasonable, or else the development of the model is likely to proceed on a weak foundation. Justification is supplied where considered necessary in the following discussion.

1. Essentially fixed inputs for a given type aircraft:
   a. Time required to climb to altitude and proceed to assigned station after launching.
   b. Time to return from station and be recovered by carrier.
   c. Endurance time (either on station or total) for a given loading.
   d. Average turn around time to ready for next launching aboard carrier.
   e. Percentage of deck space occupied by each aircraft.
2. Inputs that must be estimated:
   a. The probability that any given aircraft will incur equipment failure while airborne requiring non-routine maintenance upon return to carrier.
   b. The probability that any given aircraft will experience equipment failure just before, during, or after launching that will cause the flight to abort, and the aircraft to undergo non-routine maintenance. (The probability of a "deck dud").
   c. The number of maintenance "spots" available for repairing

aircraft on the hangar deck.

d. The average time to repair failed aircraft and return them to a fully ready status. This may be given as a mean repair rate.

e. Number of aircraft becoming ACCP during the operation.

f. Percentage of aircraft complement available at beginning of operation in a fully ready status.

3. Arbitrary inputs for a given type aircraft:

a. The number of stations desired filled continuously during the operation.

b. Intra-carrier operating doctrine, i.e., alternating launches by each CVA or simultaneous launches of approximately half the required number.

The fixed inputs require only minor discussion, since they may be obtained from aircraft performance and characteristics charts, and from deck space allocations mentioned in the assumptions. If the aircraft is still in the design or development stage, planned values may be used initially and revised when actual performance data are available.

The estimated inputs must be fixed or some realistic range of values given. For operational aircraft, statistical information might be used in estimating the inputs, but for future types or models, only estimates and drawing board specifications are available. Stanford Research Institute has done considerable work in attempting to determine a range of values for these inputs for specific types of aircraft, using manufacturer's specifications, Bureau of Weapons estimates, and aircraft squadron operational experience. The sensitivity analysis discussed later in this paper will cover in more detail the effects of varying these inputs. It is the author's belief that a series of computer runs must be made

for each set of fixed and arbitrary inputs, covering a range of values for the estimated inputs.

CHAPTER V

THE MODEL — DESCRIPTION

The Air Operations model described herein takes certain predetermined inputs and operates on them to produce one output which is a direct measure of effectiveness of the air operations system for one particular type of aircraft. The output is presented as a probability distribution over the number of aircraft that can be maintained on station for a given length of time, given these inputs. Although the model is probabilistic in nature, it is in no way random or "Monte Carlo"; it produces an analytical solution for the probability distribution. Therefore, if reasonable confidence can be placed in the inputs, the results obtained will reflect the accuracy of the model in simulation of the operations system in question.

Basically, the mathematical model is formulated as a specific type of stochastic process called a Markov chain [1][3][4]. Suppose we examine at specific points in time, t, a process that proceeds continuously in time. Let the outcome (state) at each observation time be one of a given fixed set of outcomes E_k $(k = 1, 2, \ldots n)$. Let us assume that when we started, there was a probability distribution over the possible states, called $Q^{(t_o)} = \left[q_1^{(t_o)}, q_2^{(t_o)}, \ldots q_n^{(t_o)}\right]$, so that at the start the probability of being in state E_k will be $q_k^{(t_o)}$.

Let us define a conditional probability p_{ij} as the probability that at observation time $t+1$ outcome E_j was observed, given that at observation time t, outcome E_i was observed. If we arrange these conditional probabilities in a square matrix P called a matrix of transition probabilities, it will have the following properties:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdot & \cdot & \cdot \\ p_{21} & p_{22} & p_{23} & \cdot & \cdot & \cdot \\ p_{31} & p_{32} & p_{33} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

1. The elements are non-negative $(p_{ij} \geq 0)$ for all i and j.

2. The row sums will be unity $\left(\sum_{j=1}^{n} p_{ij} = 1 \right)$ for all i.

A matrix satisfying the above conditions is called a stochastic or Markov matrix. A Markov chain or process is completely defined by a Markov matrix of transition probabilities P and a column vector $Q^{(t_o)} = \left[q_1^{(t_o)}, q_2^{(t_o)}, \ldots q_n^{(t_o)}\right]$ which gives the probability distribution over the states at the start. Note that the dependence of the Markov chain extends over one observation period only.

To establish the states E_k for the model, let us consider two random variables defined as follows:

a. $X_a(t)$ = the number of a/c flying at time t, not having flown in the previous launch-to-launch interval.

b. $X_b(t)$ = the number of a/c in or awaiting maintenance at time t.

Then if we consider the vector $\left[X_a(t), X_b(t)\right]$ as a pair of random variables, we will have a bivariate stochastic process with the possible states E_k characterized by pairs of values [2]. If we have N total aircraft of a given type aboard the carrier, and we wish to maintain A aircraft on station, the range of values of E_k will be from (0,0) to (A,N) or $E_k = (x,y)$; $0 \leq x \leq A$, $0 \leq y \leq N$.

The initial probability distribution $Q^{(t_o)}$ over the states E_k is

simple to describe, but determination of the specific distribution may not be easy. Since it is assumed that a task force will anticipate impending operations and enter an objective area with as few aircraft in maintenance as possible, the most likely value for $X_b(t_o)$ is zero. It is also assumed that aircraft operations will not commence until entry into the objective area, at which time full cycling will start. Therefore the initial distribution $Q^{(t_o)}$ is assumed to be $q_k^{(t_o)} =$ one for $E_k = (0,0)$ and $q_k^{(t_o)} =$ zero otherwise, but the model has provisions for considering any other possible initial conditions at the discretion of the operations planner.

Let us define a typical a/c operating cycle as a unit interval of time. Observations of the process are made at the initial launch and at successive unit intervals of time (launch times). To develop the conditional probability, p_{ij}, of the system going from state E_i to state E_j in a unit interval of time, let t be a typical launch time. Launching operations commence at time t and continue to time t + e. At time t + T, aircraft from the previous launch are recovered. At time t + 1, the succeeding group of aircraft is launched, and the cycle repeats. Therefore, the unit of time is the launch-to-launch interval, t to t + 1.

In the interval t to t + e, aircraft are launched with some probability of success. Those that are not successfully launched enter maintenance. In the model, this launching process continues until the desired number of aircraft (A) are successfully launched, or until there are no more available aircraft on deck. Although this time interval is of finite length it is small compared to the unit of time and it is treated as a point in time by letting e become zero.

Commencing at time t + T, the aircraft from the previous flight are

recovered. Again this interval of recovery from $t + T$ to $t + T + e$ is idealized as a point in time. The consequences of assuming that the launch and recovery intervals are of zero time duration will be discussed in more detail in the sensitivity analysis. When the aircraft are recovered, some may have experienced equipment failure in flight characterized by a failure probability described in assumption 12, and will also enter maintenance. Those that do not have failures will be rearmed, refueled, and be made ready for succeeding launches.

During the unit time interval, maintenance will be performed on those aircraft in a non-ready status, and a certain number of aircraft will be repaired as noted in assumption 10. There is a chance that the entire unit time interval will not be available for maintenance. This possibility is discussed in the sensitivity analysis.

To summarize, we start with some initial distribution, $Q^{(t_0)}$, over the possible states E_k at time t_0; launch, recover, and repair aircraft in a unit time interval; then repeat the process for the succeeding intervals until the end of the operating period (see figure 1, pg. 16). If we know the transition probabilities for all states of the system, we may form the transition matrix P, the elements $p_{(\alpha, i), (\beta, j)}$ being the probabilities of going from state (α, i) to state (β, j) in unit time for all possible states, and thus determine the probability distribution over the states $\{(\beta, j)\}$ at the start of any succeeding cycle as follows:

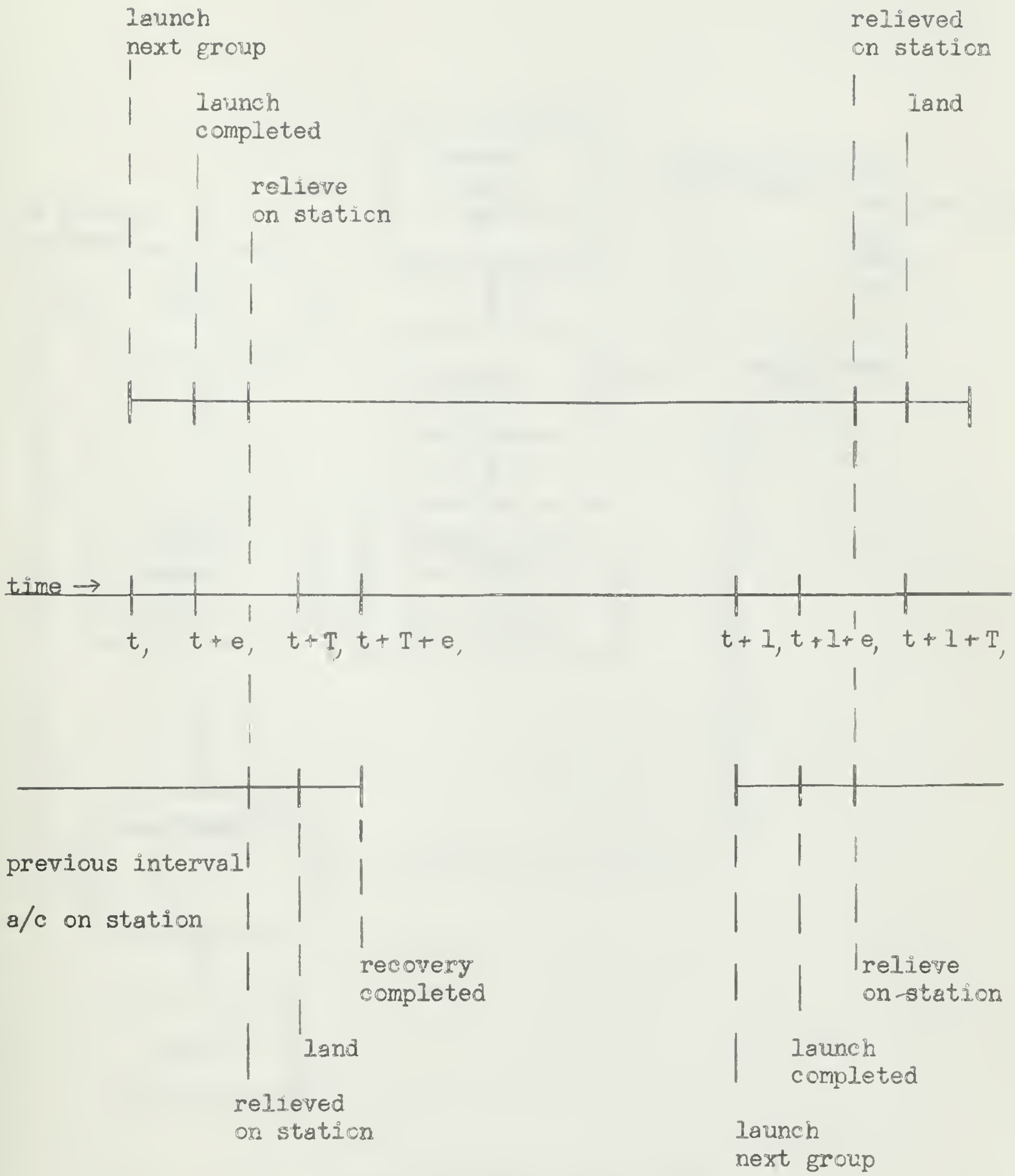
$$Q^{(t+1)} = \Pr\left[X(t+1) = (\beta, j)\right] = Q^{(t)} \cdot P .$$

Finally, the desired measure of effectiveness may be obtained at any unit time t (i.e., the start of any cycle) by summing appropriate maintenance state probabilities, i.e., the probability that α a/c are in the

$$\text{air} = \Pr\left[X_a(t) = \alpha\right] = \sum_{i=0}^{N} \Pr\left[X(t) = (\alpha, i)\right].$$

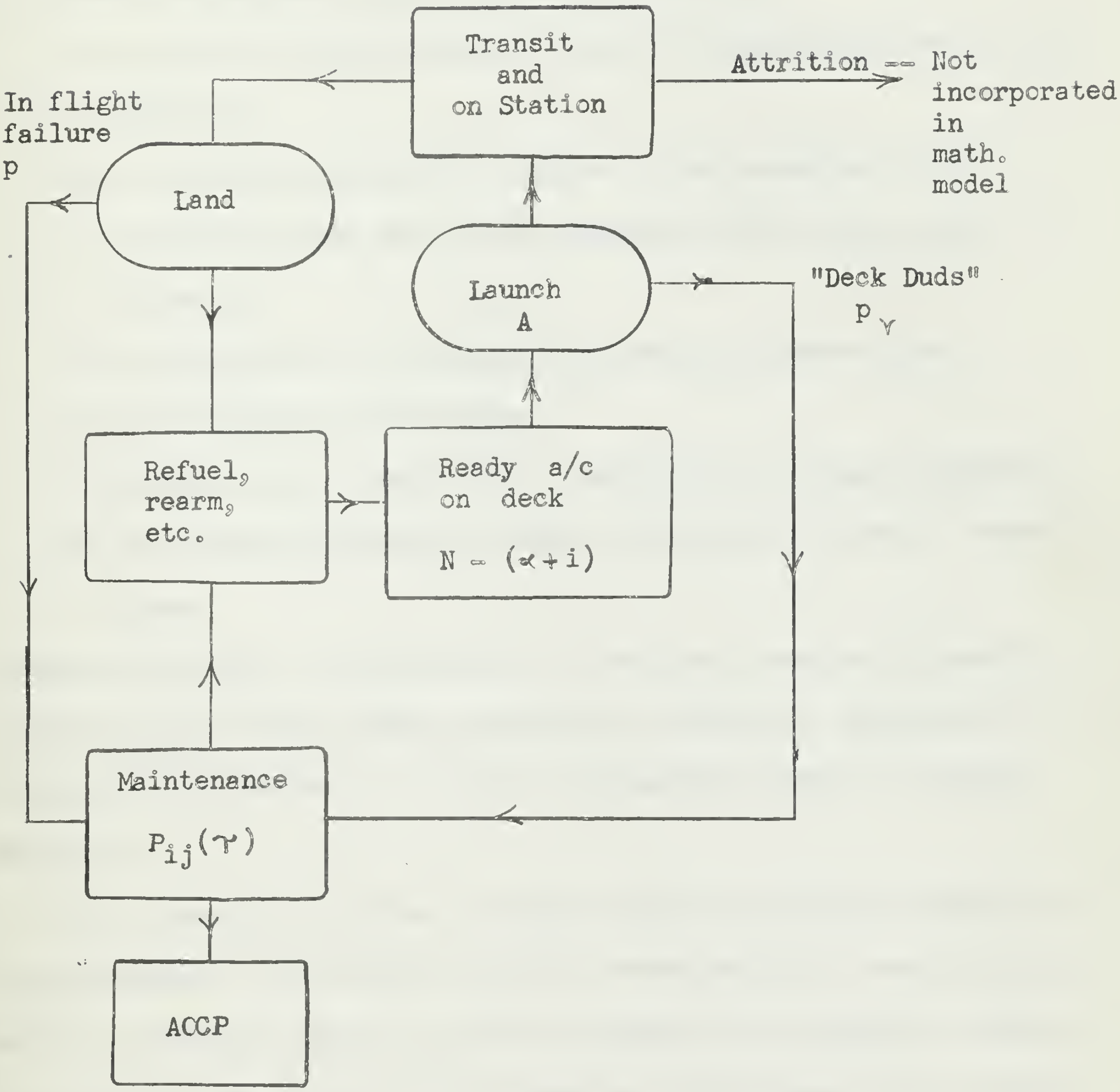
This is the probability that α aircraft are launched regardless of the number in maintenance.

The transition matrix P is complex in nature and the development of its elements is described in appendix A.



Basic Diagram of the Operating Cycle

Figure 1



Block Diagram of the Model

Figure 2

CHAPTER VI

RESUMÉ OF COMPUTER PROGRAM

The computer program of this mathematical model was written for the CDC - 1604 digital computer. The following items are available at the U.S. Naval Postgraduate School, Monterey, California:

a) IBM Cards of the program in CDC CO - OP Assembly Routine (AR) language.

b) A complete line printer printout of the assembled program.

c) A punched paper tape of the program in "AR" format, ready for assembly.

d) Description of the program and Operating Instructions.

e) Flow charts of the program.

f) Sample copy of output for a specific set of input parameters.

g) The complete program in Binary Coded Decimal format on magnetic tape.

Readers interested in obtaining any of the above items or information concerning them should address inquiries to Secretary, Department of Mathematics and Mechanics, U.S. Naval Postgraduate School, Monterey, California.

The program was written in several parts to facilitate checking out and "debugging". Initially, the input parameters for a specific run or series of runs are placed in a table in memory. The required values of $P_{ij}(\tau)$ are then computed in two separate subroutines, one for direct evaluation of equations 3 and 4, pg. 29, and the other for solution of the system of differential equations (equation 1, pg.32) using numerical integration. The values obtained are stored in a table in com-

puter memory, using a parallel table storage of the values (i,j) to facilitate retrieval and retain the accuracy of the computed values. Next, the values of $\pi_\alpha(m)$ and γ_{fgh} are computed, using subroutines included in the program, and stored in memory using the same procedure described above for $P_{ij}(\tau)$. The program then generates the transpose of the transition matrix, P', and stores the elements in memory in consecutive order. The matrix is generated in transposed form to facilitate indexing and future matrix multiplication. A jump is then made to a routine that sums each row of the matrix and transfers these sums in floating point decimal format to magnetic tape for printout to check the accuracy of computations.

The initial probability distribution vector $Q^{(t_o)}$ is then formed, the matrix product $Q^{(t_o)} \cdot P = Q^{(to+1)}$ is generated, and the measure of effectiveness is obtained from this new vector (by summing over all values of i for fixed α) and transferred to magnetic tape in floating point decimal format for subsequent printout. This new vector $Q^{(t_o+1)}$ then replaces the original vector $Q^{(t_o)}$ and the next matrix product is formed. This process continues until the end of the operating period is reached; then if investigation of other initial distributions is desired, the above process is repeated. If not, computation of the measure of effectiveness is completed, and (at the discretion of the operator) either the program stops, or the last value of the measure of effectiveness $Pr\left[X_a(t) = A\right]$ is checked to see whether it is less than some preset value (presently .9). If not, A is increased by one, and computation is resumed, omitting certain parts of the program that do not require re-running. Finally, when the above condition is satisfied, computation stops.

There is a separate section of the program that may be used if it is desired to conduct a sensitivity analysis similar to that described in this thesis. Initial values are inserted and computation is begun in a different location. Parameters are varied within the same range of values shown in Appendix C until the series of runs is completed. Results are printed in consecutive order on magnetic tape for printout on a line printer.

The computer program with all subroutines and storage cells except those for decimal conversion and output is approximately 3000 two instruction words in length. Essentially all computations are done using floating point arithmetic. There are 28,160 cells (of a total of 32,768) reserved for matrix elements, restricting the values of A and N in that $(A + 1)(N + 1) \leq (28{,}160)^{\frac{1}{2}} \approx 167$. If the values of A and N should not satisfy this restriction, it would be necessary to re-program the model, perhaps storing portions of the matrix on magnetic tape while generating the remainder.

The size of the matrix is the major factor determining running time for the program. For an A of two and an N of five, running time is approximately two minutes, while for an A of six and an N of 20, this time increases to approximately 35 minutes. Minor attempts to rewrite portions of the program and reduce the length of time required for computation have not significantly lowered these values. Although the author is not an experienced programmer, it appears that the number of arithmetic operations required for generating each element of the matrix cannot be reduced, and these approximate times must be accepted. However, it is emphasized that the cost of 35 minutes of computer time compares favorably with the cost of 72 hours of task force time.

CHAPTER VII

SENSITIVITY ANALYSIS

The model that has been created to transform inputs into outputs does not appear susceptible to a mathematical sensitivity analysis due to its complexity. However, a lengthy program of variation of input parameters was conducted to compare numerically the resulting variations in output. To limit the analysis to inputs which were essentially estimates, the following values were fixed for a simulated type aircraft:

A: Four a/c desired on station   a/c = aircraft

N: 13 a/c available aboard CVA

D: Six maintenance spots available

Cycle length: Four hours

γ: .2 of cycle length

Duration of operation: 72 hours

The following inputs were varied within the ranges shown:

The probability of equipment failure in flight (p): 0(.05).5 or 0(.1).5.

The probability of a deck dud (P_γ): 0(.03).21 or .0(06).18.

The mean repair rate (λ): 4(4)20.

The fraction of cycle time available for maintenance 1.0(.2).8.

Tabulated and graphic results of these runs are shown in Appendix C and D and are to a large extent self explanatory. They illustrate how variation of each parameter affects the measure of effectiveness.

Although inputs may be varied at the discretion of the user, it is envisioned that basically two types of group runs of the computer model would be made. They are:

1. Predetermined fixing of the probabilities and maintenance values and variations of A, N, and Cycle Length (and with it Υ), to determine the optimum loading, stationing, and cycling of a particular type of aircraft.
2. Fixing the number of aircraft available, their stationing and cycling, and varying the maintenance values and the probabilities of failure, as has been done for the results presented herein.

The first type of group runs was conducted for SRI using current and future fleet aircraft parameters in order to determine optimum loading stationing and cycling policies for these types of aircraft. The results cannot be included due to their classification, but realistic output values were obtained.

For any mathematical abstraction of a real world process, the question of goodness of fit of the model to the actual situation is the basic test of the value of the model. As mentioned in the introduction, this question has not been answered directly; however, the answers obtained for any given situation "seemed reasonable". One of the functions of an operations analyst is to recommend or arrange for sufficient collection of data, usually to provide quantitative values for a statistical analysis of a problem. Therefore, it is the author's recommendation that if this model is used by the Navy (and he hopes that it will be), a program for collection of required data be established for future large-scale carrier operations. Perhaps some of the required inputs are available from already existent reports, and the remainder might be obtained by modifying their format. A proposed data collection form for inputs required by this model is given in figure 3.

MONTH ________ YEAR ____ SQUADRON ______ TYPE A/C ______

Day	1	2	3	. . . . .	31
1. # a/c aboard					
2. # a/c in ready status					
3. # a/c AOCP					
4. # sorties					
5. Average duration of each flight					
6. # of aborted flights, or a/c downed just prior to or during launch					
7. # a/c down for routine maintenance or modifications					
8. # a/c entering non-routine maintenance this day a. upon return from flight					
b. from a ready status					
9. # a/c leaving non-routine maintenance this day					
10. Time required to repair each of these a/c (list time in hrs. for each a/c)					
11. # of man hours required for repair (list for each a/c)					

Sample Maintenance Report Form

Figure 3

Variation of the length of time available in any one cycle for maintenance was considered for two reasons. First, it appeared likely, although not mandatory, that maintenance work might stop during launching and recovery operations. Secondly, the assumption that launch and recovery intervals could be idealized as points in time would have to be modified if maintenance work was halted in these intervals. For the four hour cycle length chosen for the sensitivity runs, a significant degradation of maintenance time of 48 minutes was chosen to show the effects of continuous as opposed to interrupted work. If the carrier were operating only this or a similar type of a/c and maintained four hour launch intervals, 48 minutes appears excessive. However, if other a/c types of limited endurance were being launched and recovered, this value is reduced to about 15 minutes per launch-recovery operation. Since maintenance work might also be interrupted for relocation of a/c on the hangar deck, it seemed desirable to consider this longer time. On the other hand, for a cycle length of 90 minutes, this degradation results in a loss of 18 minutes of time for maintenance work, again a likely value.

The effects of this loss in repair time may be noted in figures 17 and 18, Appendix D. As shown, the effect of not having the entire interval available is essentially negligible for a fairly short mean repair rate and smaller failure probabilities. As the average length of time required to repair each a/c increases, the fraction of time the maintenance system is not working becomes more significant, certainly an expected result. The same effect is evident when the failure probabilities are increased.

Fortunately for a mathematical model of this size and complexity,

there are certain built-in checks of the correctness of the computed answers. Although computational subroutines were checked before use to ensure their exactness, the row sums of the transition matrix P give an adequate check of the correctness of a major portion of the program. At no time during the computation did these row sums differ from one by more than 10^{-3}, and in most cases this difference was less than 10^{-8}. The small errors, when appearing, were traced to the numerical integration subroutine for $P_{ij}(\tau)$, and appeared to be the result of inherent roundoff errors in the computer, when converting fractions to fixed length octal numbers. Considering the number of arithmetic operations performed in the program, the exactness is certainly satisfactory. A second check is obtainable from the measure of effectiveness, in that the sum of the probabilities of launching zero up to A a/c should be one. Again, the same exactness indicated above was obtained. Note that the computer program produces "exact" answers that are arithmetically correct, as opposed to "accurate" answers, which to the author imply correctness in the sense of duplication of the real world system. This accuracy has been discussed elsewhere in this paper.

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APPENDIX A

DEVELOPMENT OF THE TRANSITION MATRIX P

The elements of the transition matrix, p_{ij}, are the probabilities of transition from state E_i to state E_j in the unit time interval. Considering a typical unit interval, at the beginning of the interval there is a probability distribution over all the possible states. Let us define γ_{fgh} as the probability that of f aircraft on a carrier deck ready to fly, g will be launched and h will be duds requiring repair, each a/c having the same chance of successful launch characterized by a success probability s, which is one minus the probability of a "deck dud", p_γ. Since each a/c launch is in itself an independent event, and we desire to obtain A successful launches, γ_{fgh} will have the following values:

1. 0	if $g > A$
2. 0	if $g + h > f$
3. 0	if $g < A$, $g + h < f$
4. $\binom{f}{g} s^f (1 - s)^{f-g}$	if $g < A$, $g + h = f$
5. $\binom{g+h-1}{h} s^g (1 - s)^h$	if $g = A$.

The reasoning for these values is as follows:

1. It is desired to launch no more than A aircraft.
2. It is impossible to launch and send into maintenance more a/c than the number available.
3. If the required number A are not obtained and there are still ready a/c left, launching will continue.
4. The standard binomial "success-failure" expression for independent trials, where all ready a/c are used, but the desired

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number A are not launched.

5. The standard negative binomial expression for obtaining g successes in $g + h - 1$ trials, i.e., the desired number are launched without utilizing all available ready aircraft.

Considering next the time $t + T$ when say α a/c return and are recovered, we must determine the probability that say, m of these a/c require repair upon landing and enter maintenance, called $\pi_\alpha(m)$. Since each flight is an independent event as far as failures are concerned, $\pi_\alpha(m)$ will be a standard binomial distribution, i.e.,

$\pi_\alpha(m) = \binom{\alpha}{m} p^m (1 - p)^{\alpha - m}$, where p is the probability of equipment failure in flight. Those a/c that do not fail are refueled, rearmed, and prepared for succeeding flights.

Turning to maintenance procedures, let us define $P_{ij}(\tau)$ as the probability of repairing $(i - j)$ a/c in time τ, given i a/c in or awaiting maintenance at time t. An alternate definition would be the probability of having j a/c in or awaiting maintenance at time $t + \tau$, given i at time t. Under the assumptions given in part 10, a pulsed input, exponential holding time, multiple server queue [5] may be developed as follows: let D be the number of maintenance "spots" available. These are service channels in queue terminology. Each of these service channels will have an identical mean service rate, λ, and an exponential service or holding time. OEG Report 585 (Confidential) verifies the exponential service time for maintenance of aircraft aboard carriers during the Korean War, the latest information available for large scale operations.

Then for each channel occupied, the probability that it remains

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occupied throughout a length of time τ is $e^{-\lambda\tau}$, and the probability that it becomes free (repairs the a/c) in this time is $1 - e^{-\lambda\tau}$.

The queue equations which were arrived at independently are identical to those given in [6]. They are:

$$1. \quad \frac{dP_{i,n}(t)}{dt} + n\lambda P_{i,n}(t) = (n+1)\lambda P_{i,\,n+1}(t) \quad \text{for } 0 \leq n \leq D,$$

$$2. \quad \frac{dP_{i,n}(t)}{dt} + D\lambda P_{i,n}(t) = D\lambda P_{i,\,n+1}(t) \quad \text{for } n \geq D,$$

with $P_{ij}(0) =$ one if $i = j$, zero otherwise. If there are fewer aircraft awaiting or in maintenance than there are "spots", there is no queue, and the resulting distribution of $P_{ij}(\tau)$ is binomial, since each spot is considered to work independently of the others. In this case,

$$3. \quad P_{ij}(\tau) = \binom{i}{j}(1 - e^{-\lambda\tau})^{i-j}\, e^{-j\lambda\tau}.$$

Naturally, if $i < j$, $P_{ij}(\tau)$ is zero since there is no way for the pulsed queue to increase with no input in the interval of repair considered.

Equation 2 above may be solved in closed form to give:

$$4. \quad P_{i,n}(t) = \frac{(D\lambda t)^{i-n}}{(i-n)!}\, e^{-D\lambda t} \qquad \text{for } i \geq n \geq D.$$

The solution of the system of equations given in 1 above is discussed in Appendix B. Since the work involved in producing and evaluating a closed form solution is at least comparable in time to that required for solution by numerical integration, a program for solution using numerical integration by the CDC-1604 computer was written and used, and provided completely satisfactory rapid answers.

To summarize the values of $P_{i,j}(\tau)$ then,

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P_{ij}(\tau): \begin{cases} 0 & \text{if } j > i \\ \binom{i}{j}(1 - e^{-\lambda\tau})^{i-j}\, e^{-j\lambda\tau} & \text{if } j \le i \le D \\ \dfrac{(D\lambda\tau)^{i-j}}{(i-j)!}\, e^{-D\lambda\tau} & \text{if } i \le j \le D \\ \text{Numerical solution of equation 1} & \text{if } j < D < i. \end{cases}
$$

check

To generate the transition matrix P, we must consider all the possible events taking place in the unit time interval as follows: At time t, we start with α a/c in the air and i a/c in maintenance; a/c are launched and enter maintenance at time t + e; a/c are repaired from time t + e until time t + T, when a/c are recovered and some may enter maintenance; a/c are repaired in maintenance until the end of the unit cycle, t + 1. Then in words, the probability that there are β a/c flying and j a/c in maintenance at t + 1, given α a/c flying and i in maintenance at t is the probability that the system changes from state (α, i) to state (β, j) in unit time. Each p_{ij} is composed of the triple sum over all possible values of the product of four event probabilities, i.e.

$$
P_{(\alpha,i),\,(\beta,j)} = \sum_{k,l,m} \gamma_{N-(\alpha+i),\beta,l} \cdot P_{i+l,k}(T) \cdot \pi_\alpha(m) \cdot P_{k+m,j}(1-T).
$$

Explanation of the indices is as follows:

a. $\gamma_{N-(\alpha+i),\beta,l}$. There are $N-(\alpha+i)$ a/c available at time t, of which β are launched, and any number, l, from 0 to $N-(\alpha+i)$ $-\beta$! enter maintenance.

b. $P_{i+l,k}(T)$. Initially, there were i a/c in maintenance, to which l a/c are added when launching occurs. In time T, the maintenance queue diminishes in length to k as a/c are repaired.

c. $\pi_\alpha(m)$. At time T, m of the α a/c flying enter maintenance,

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where m may range from 0 to α.

d. $P_{k+m,j}(1-T)$. At time T, the maintenance queue receives m additional a/c, and in the remainder of the interval, $(1-T)$, the queue diminishes in length to j a/c. The limits on k are from zero to the total number of a/c that could enter maintenance in the interval considered, $i+1$.

The transition matrix is of order $(A+1)(N+1) \times (A+1)(N+1)$ since all possible combinations of (α, i) are allowed. However, null states occur in the following cases:

1. $\alpha + i > N$;
2. $\beta + j > N$;
3. $\alpha + i + \beta > N$.

In each of these cases, it is impossible to use more aircraft than there are available, and consequently all these probabilities are zero.

APPENDIX B

SOLUTION OF THE QUEUEING EQUATIONS

The equations of balance for the pulsed input queue (equations 1 and 2, Appendix A) are

$$1.\quad \frac{dP_{i,n}(t)}{dt} + n\lambda P_{i,n}(t) = (n+1)\lambda P_{i,n+1}(t) \qquad \text{for } 0 < n < D,$$

$$2.\quad \frac{dP_{i,n}(t)}{dt} + D\lambda P_{i,n}(t) = D\lambda P_{i,n+1}(t) \qquad \text{for } n \geq D,$$

with $P_{ij}(0) =$ one if $i = j$, zero otherwise. As noted in Appendix A, equation 2 is solvable in closed form and this solution is given there. When $n = D - 1$, equation 1 above becomes

$$3.\quad \frac{dP_{i,\overset{D}{\cancel{D-1}}}(t)}{dt} + (D - 1)\lambda P_{i,\overset{D}{\cancel{D-1}}}(t) = D\lambda P_{i,\underset{+1}{D}}(t).$$

From Appendix A, $P_{i,D}(t) = \frac{(D\lambda t)^{i-D}}{(i-D)!}\, e^{-D\lambda t}$.

Denoting $P_{i,D-1}(t)$ as x, equation 3 is of the form $\frac{dx}{dt} + Qx = R$, where Q and R are functions of t only or constants. This is a linear differential equation and its solution for x is

$$x\, e^{\int Q\,dt} = \int R\, e^{\int Q\,dt}\, dt + C,$$ C a constant of integration to satisfy the initial or boundary conditions. Substituting for Q and R and simplifying, this becomes

$$x = e^{-(D-1)\lambda t} \quad \frac{D\lambda (D\lambda t)^{i-D}}{(i-D)!} \quad e^{-D\lambda t} \quad e^{(D-1)\lambda t}\, dt + C$$

which eventually reduces to

$$x = e^{-D\lambda t}\, D^{i-D+1} \left[e^{\lambda t} - \sum_{j=0}^{i-D} \frac{(\lambda t)^j}{j!} \right]$$ as a solution

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for $P_{i, D-1}(t)$. Note that the original limits of i and j for $P_{ij}(t)$ were $j < D < i$ for this system, so that the first value of i, D+1, gives a solution of

$x = e^{-D\lambda t} D^2 \left[e^{\lambda t} - 1 - \lambda t \right]$. To give a numerical example, suppose D = four maintenance spots, λ is one half the cycle length in time, and t is one fourth the cycle length in time; then the probability that the maintenance queue would change from five (D+1) to three (D-1) a/c awaiting or in maintenance in one fourth the cycle is

$P_{5,3}(\frac{1}{4}) = e^{-\frac{1}{2}} (4)^2 \left[e^{1/8} - 1 - 1/8 \right] = .079$ to slide rule accuracy.

For the next smaller value, n = D - 2, equation 1 becomes

$$\frac{dP_{i,D-2}(t)}{dt} + (D-2)\lambda P_{i,D-2}(t) = (D-1)\lambda P_{i,D-1}(t).$$

Using the expression for $P_{i,D-1}(t)$ given above, the solution for $P_{i,D-2}(t)$ is

$$P_{i,D-2}(t) = e^{-(D-2)\lambda t} \lambda(D-1) D^{i-D+1} \int e^{-2\lambda t} \left[e^{\lambda t} - \sum_{j=0}^{i-D} \frac{(\lambda t)^j}{j!} \right] dt + C.$$

This integration has been carried out and an expression for $P_{i,D-2}(t)$ obtained in explicit form, as has been done for $P_{i,D-3}(t)$. Both of these expressions are lengthy and complicated, and succeeding expressions for decreasing n will be even more complex. Since the expressions must be coded for computer solution for utilization with the remainder of the model, it appeared that solution of the system of differential equations could best be handled by direct numerical integration using a subroutine of the Runge-Kutta-Gill type available for the CDC-1604 computer. The

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resulting program provided rapidly obtained accurate values for $P_{i,n}(t)$ for all requisite values of i, n, and t, using the initial values of P and P' at time zero and a delta t of .001 or .005. In addition, it is interesting to note that corresponding values tabled in [6] were verified. Accuracy of the solutions obtained is discussed briefly elsewhere in this paper.

APPENDIX C

TABLE 1

TABLES OF NUMERICAL RESULTS OF COMPUTER RUNS FOR VARIOUS INPUT PARAMETERS

1. Mean Repair Rate λ equals four hours.

a. Entire interval available for maintenance.

p \ p_γ	.00	.03	.06	.09	.12	.15	.18	.21
.6	.9953	.9902	.9822	.9705	.9545	.9338	.9082	.8774
.65	.9927	.9862	.9764	.9626	.9444	.9216	.8939	.8616
.7	.9893	.9810	.9693	.9533	.9330	.9080	.8786	.8448
.75	.9847	.9745	.9606	.9425	.9200	.8932	.8621	.8270
.8	.9788	.9665	.9504	.9301	.9056	.8769	.8443	.8082
.85	.9714	.9569	.9385	.9160	.8896	.8592	.8254	.7881
.9	.9622	.9454	.9248	.9003	.8719	.8402	.8050	.7670
.95	.9511	.9320	.9092	.8827	.8531	.8195	.7833	.7450
1.0	.9379	.9166	.8916	.8633	.8320	.7971	.7605	.7216

NOTE: Above runs were not conducted for p .55 since there is not a significant change in the measure of effectiveness.

b. .8 of interval available for maintenance.

p \ p_γ	.00	.06	.12	.18
.00	1.0	1.0	.9988	.9901
.1	1.0	.9997	.9961	.9805
.2	1.0	.9986	.9906	.9659
.3	.9993	.9955	.9807	.9455
.4	.9980	.9891	.9649	.9184

TABLE 2

2. Mean Repair Rate λ equals eight hours.

a. Entire interval available for maintenance.

p \ p_γ	.00	.03	.06	.09	.12	.15	.18	.21
.00	1.0	.9999	.9999	.9991	.9958	.9876	.9714	.9446
.05	.9999	.9999	.9993	.9969	.9904	.9771	.9544	.9207
.1	.9999	.9995	.9976	.9922	.9811	.9616	.9324	.8923
.15	.9996	.9980	.9935	.9839	.9671	.9412	.9054	.8595
.2	.9982	.9942	.9858	.9708	.9476	.9153	.8735	.8230
.25	.9946	.9869	.9731	.9520	.9224	.8840	.8373	.7833
.3	.9873	.9744	.9547	.9272	.8915	.8480	.7975	.7413
.35	.9747	.9559	.9298	.8962	.8553	.8079	.7549	.6976
.4	.9555	.9304	.8984	.8596	.8150	.7646	.7103	.6532

b. .8 of interval available for maintenance.

p \ p_γ	.00	.06	.12	.18
.00	1.0	.9998	.9919	.9502
.1	.9998	.9943	.9630	.8843
.2	.9947	.9678	.9010	.7910
.3	.9665	.9051	.8064	.6810
.4	.8975	.8055	.6915	.5678

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TABLE 3

3. Mean Repair Rate λ equals 12 hours.

a. Entire interval available for maintenance.

p \ p_r	.00	.03	.06	.09	.12	.15	.18	.21
.00	1.0	.9999	.9995	.9965	.9862	.9629	.9222	.8631
.05	.9999	.9996	.9968	.9875	.9666	.9300	.8764	.8070
.1	.9995	.9967	.9878	.9683	.9346	.8850	.8207	.7445
.15	.9963	.9873	.9684	.9363	.8896	.8292	.7575	.6783
.2	.9856	.9667	.9352	.8904	.8329	.7650	.6899	.6111
.25	.9630	.9314	.8875	.8321	.7673	.6958	.6209	.5454
.3	.9246	.8808	.8266	.7645	.6963	.6250	.5531	.4829
.35	.8701	.8170	.7567	.6915	.6237	.5555	.4888	.4251
.4	.8024	.7439	.6816	.6172	.5527	.4897	.4293	.3727
.45	.7262	.6666	.6058	.5450	.4858	.4291	.3756	.3260
.5	.6470	.5896	.5328	.4775	.4245	.3745	.3278	.2847

b. .8 of interval available for maintenance.

p \ p_r	.00	.06	.12	.18
.00	1.0	.9990	.9732	.8689
.1	.9984	.9716	.8772	.7139
.2	.9613	.8662	.7156	.5434
.3	.8344	.6919	.5370	.3937
.4	.6440	.5076	.3836	.2796

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TABLE 4

4. Mean Repair Rate λ equals 16 hours.

a. Entire interval available for maintenance.

p \ p_r	.00	.03	.06	.09	.12	.15	.18	.21
.00	1.0	.9998	.9988	.9910	.9677	.9207	.8482	.7549
.05	.9999	.9984	.9902	.9671	.9223	.8541	.7663	.6663
.1	.9977	.9882	.9642	.9199	.8543	.7708	.6759	.5770
.15	.9848	.9585	.9134	.8490	.7688	.6783	.5841	.4920
.2	.9495	.9023	.8381	.7601	.6737	.5842	.4967	.4148
.25	.8860	.8212	.7451	.6622	.5775	.4948	.4174	.3472
.3	.7979	.7235	.6443	.5643	.4869	.4143	.3483	.2896
.35	.6954	.6201	.5452	.4732	.4059	.3446	.2897	.2414

b. .8 of interval available for maintenance.

p \ p_r	.00	.06	.12	.18
.00	1.0	.9974	.9402	.7611
.1	.9936	.9264	.7565	.5368
.2	.8898	.7198	.5216	.3470
.3	.6514	.4775	.3297	.2181
.4	.4122	.2947	.2050	.1389

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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]

TABLE 5

5. Mean Repair Rate λ equals 20 hours.

a. Entire interval available for maintenance.

p \ p_r	.00	.06	.12	.18
.00	1.0	.9974	.9402	.7607
.1	.9936	.9262	.7560	.5360
.2	.8896	.7191	.5206	.3460
.3	.6504	.4764	.3288	.2175
.4	.4111	.2939	.2043	.1389

b. .8 of interval available for maintenance.

These runs were not conducted since their inclusion would not add significantly to the results presented herein.

Mean Repair Rate λ equals 4 hours

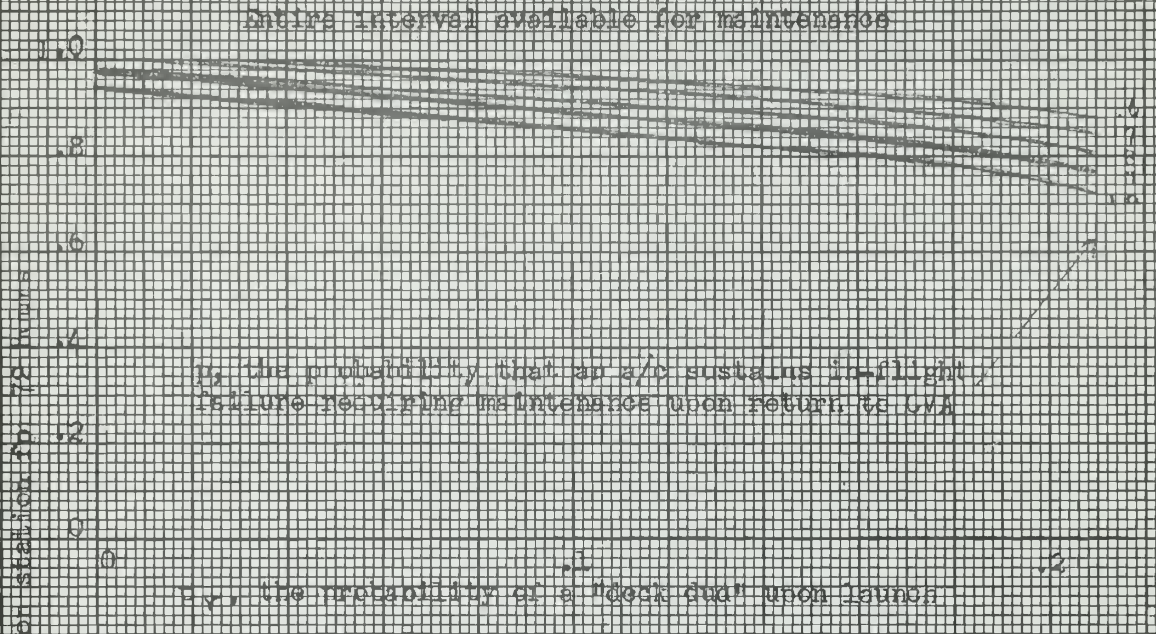


Figure 4

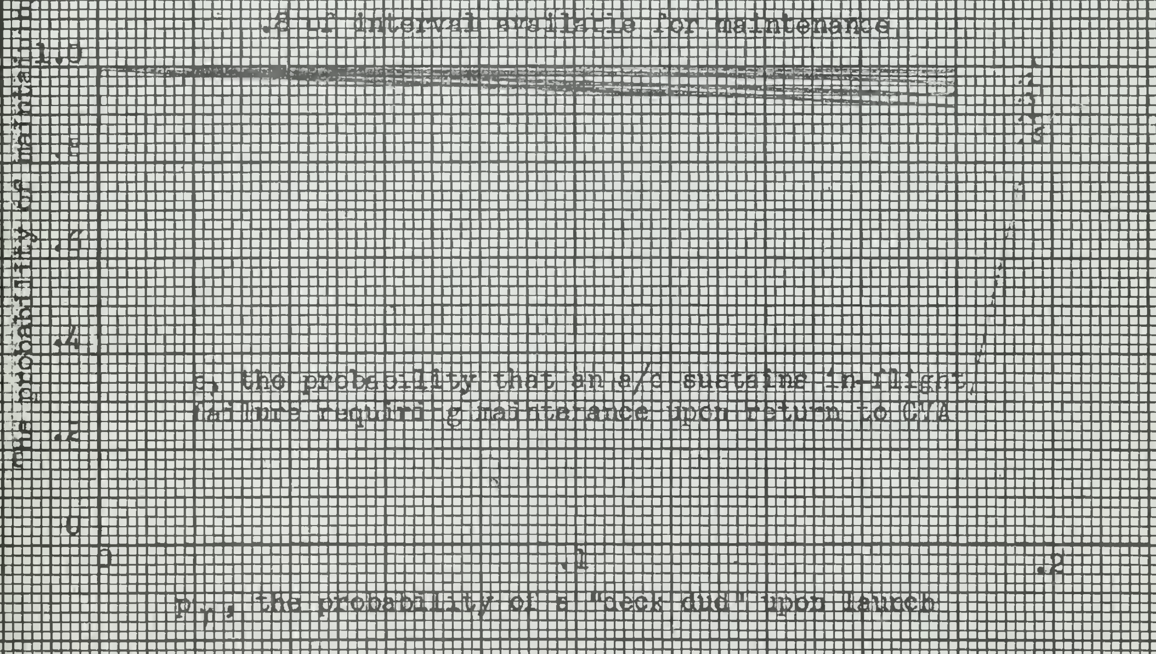


Figure 5

Mean Repair Rate λ equals 8 hours

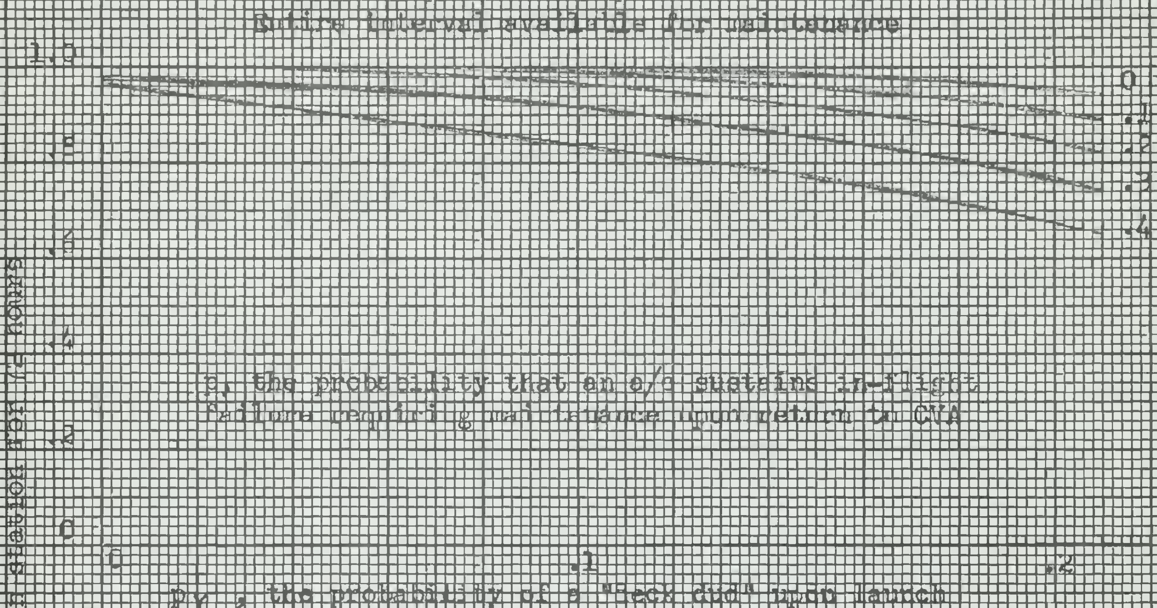


Figure 6

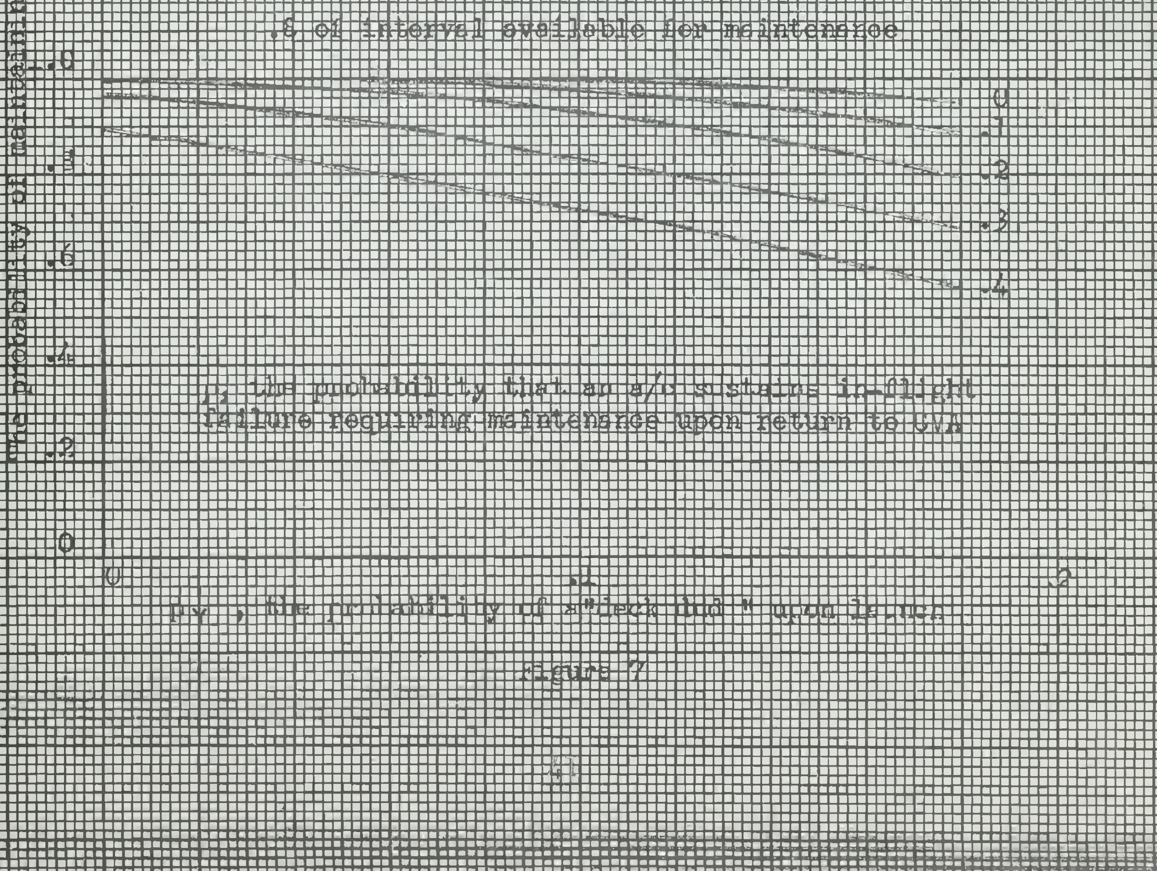


Figure 7

Mean Repair Rate λ equals 12 hours

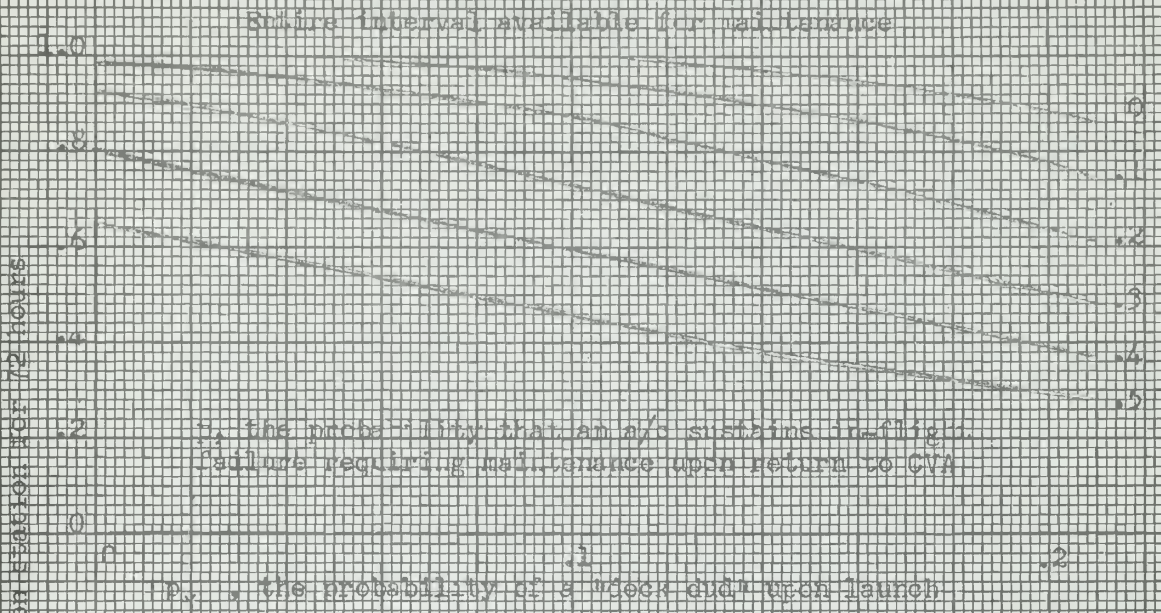


Figure 8

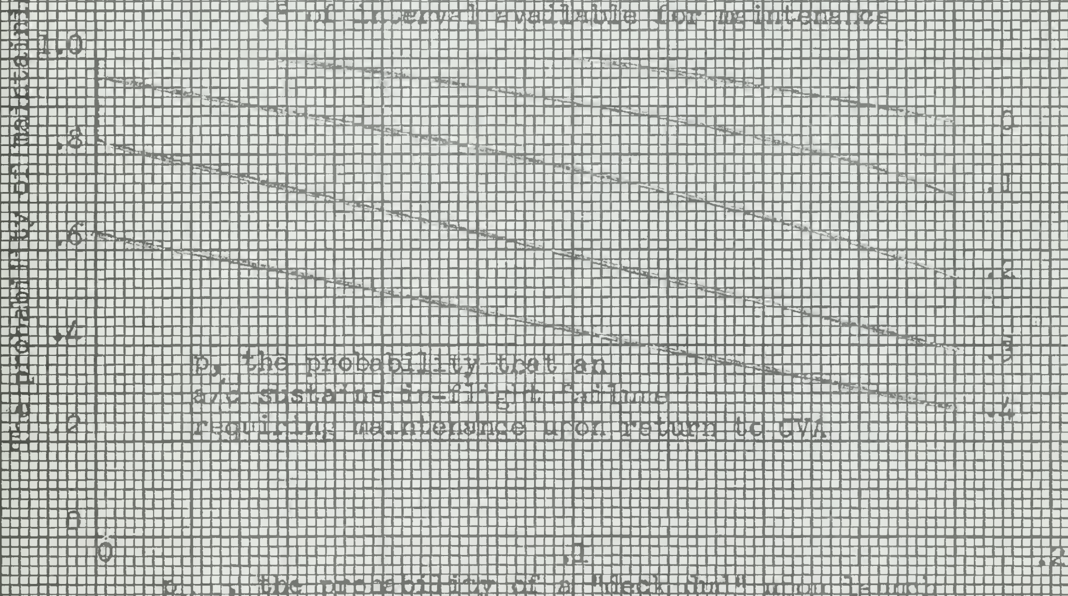


Figure 9

Mean Repair Rate λ equals 16 hours

Entire interval available for maintenance

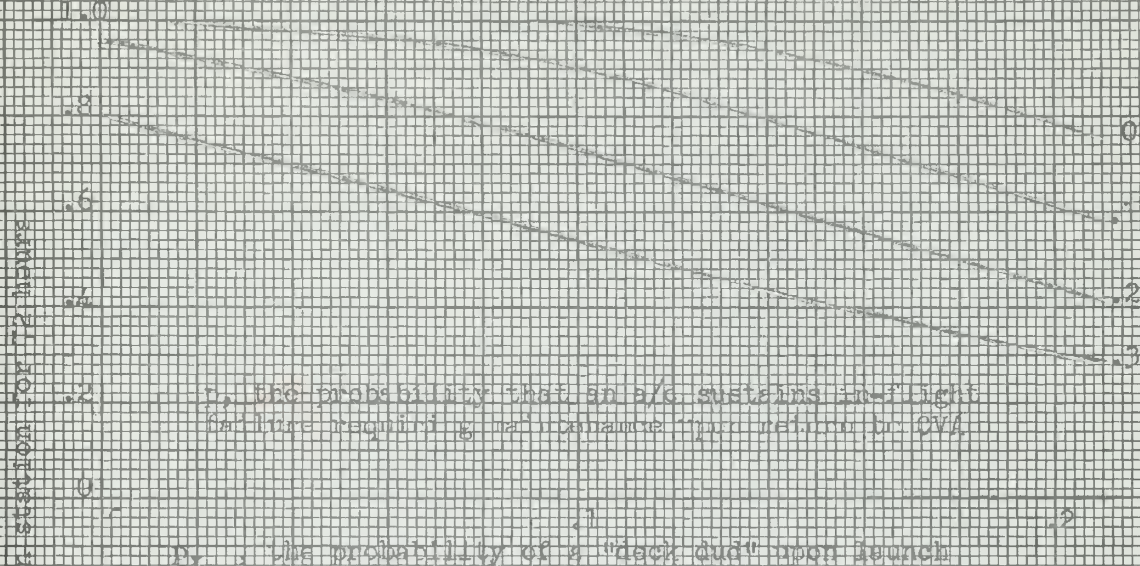


Figure 10

.8 of interval available for maintenance

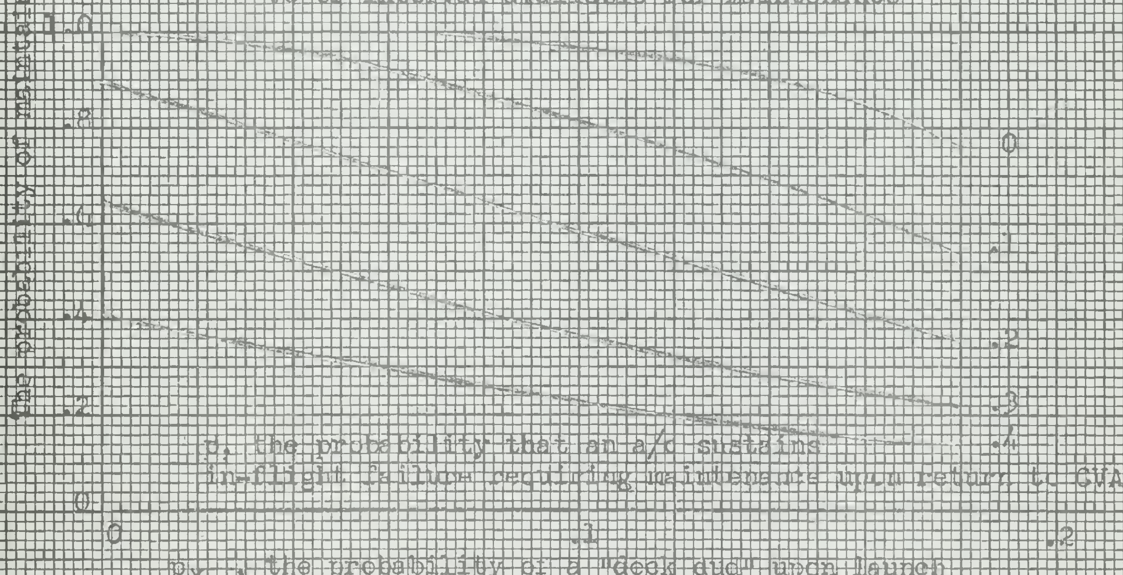
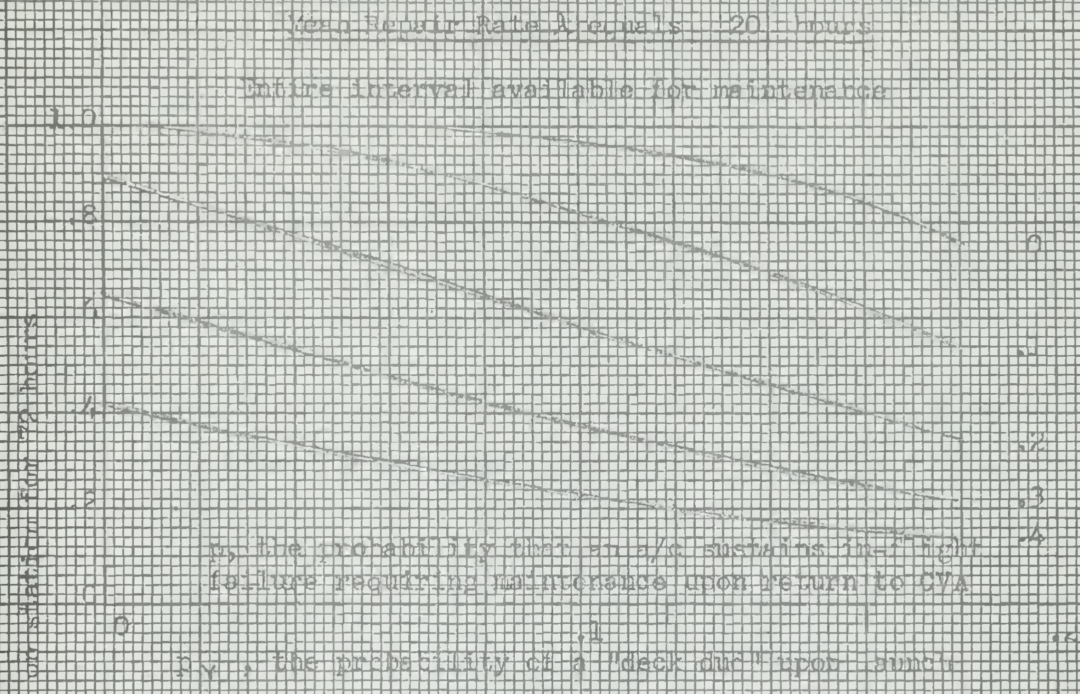


Figure 11



Mean Repair Rate λ equals 20 hours
Entire interval available for maintenance
1.0
.8
.6
.4
.2
0
0
.1
.2
.3
.4
0
.1
.2
The probability of maintaining 4 a/c on station for 72 hours
p, the probability that an a/c sustains in-flight failure requiring maintenance upon return to CVA
p_V, the probability of a "deck dud" upon launch

Figure 12

p_2, The probability of a "deck dud" upon launch equals .06

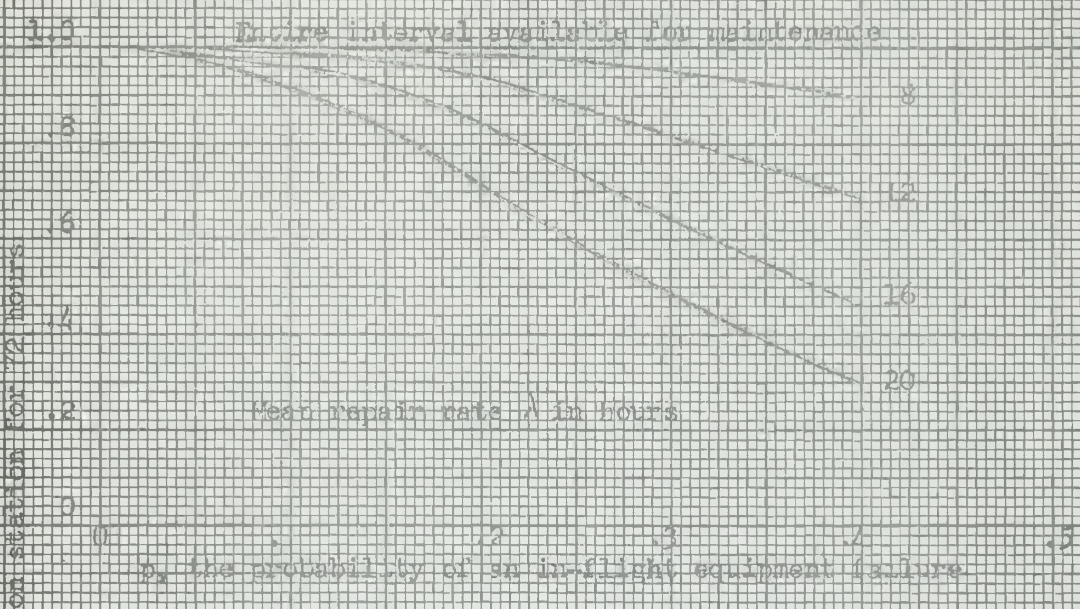


Figure 13

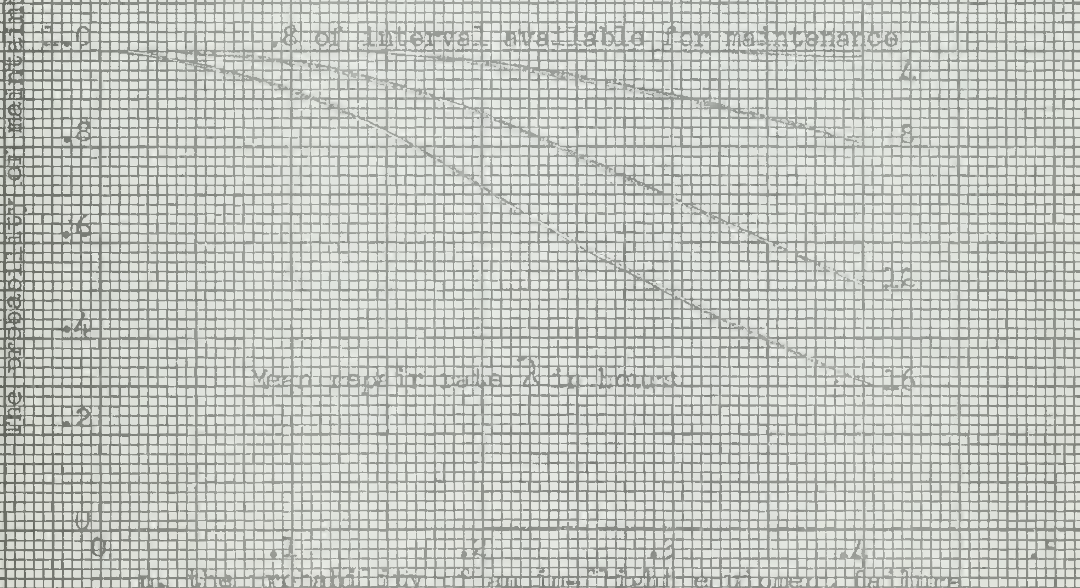
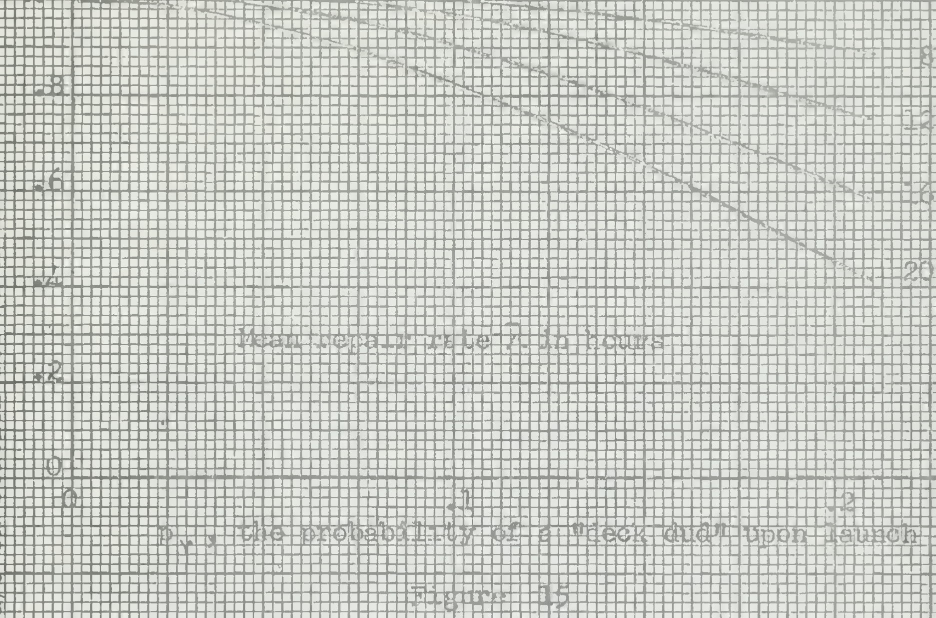


Figure 14

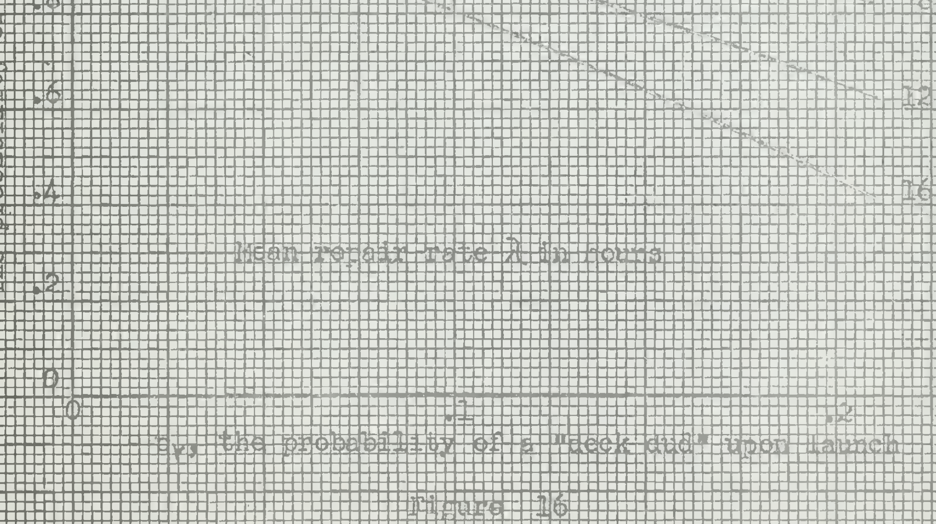
p, The probability of an in-flight equipment failure equals .1

The probability of maintaining 1 a/c on station for 72 hours



p_v, the probability of a "deck dud" upon launch

Figure 15



p_v, the probability of a "deck dud" upon launch

Figure 16

p_1, The probability of a "deck dud" upon launch equals .12

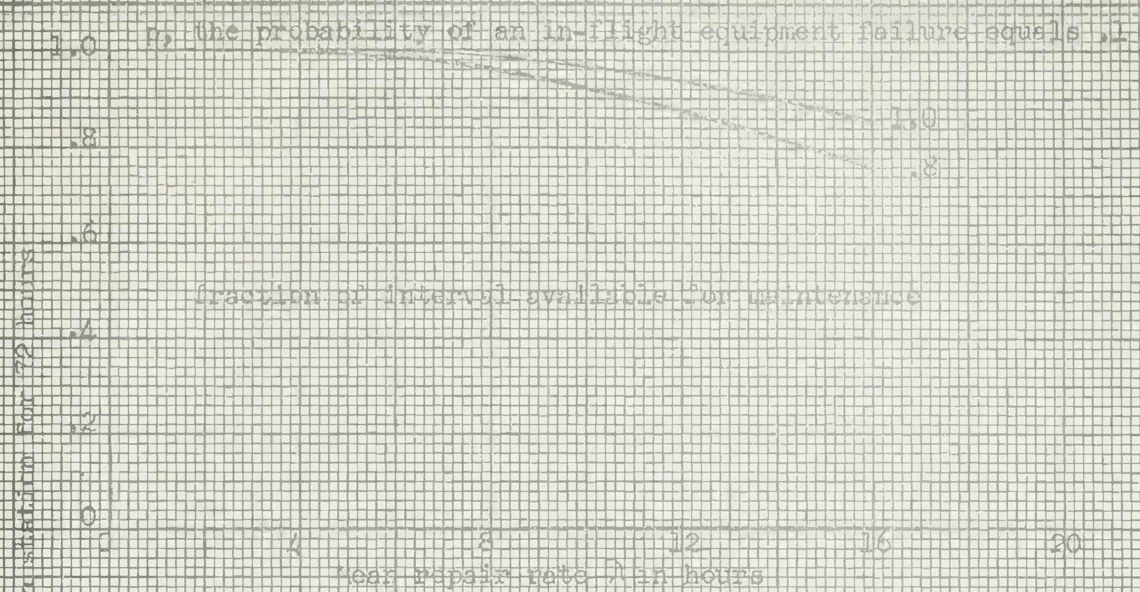


Figure 17

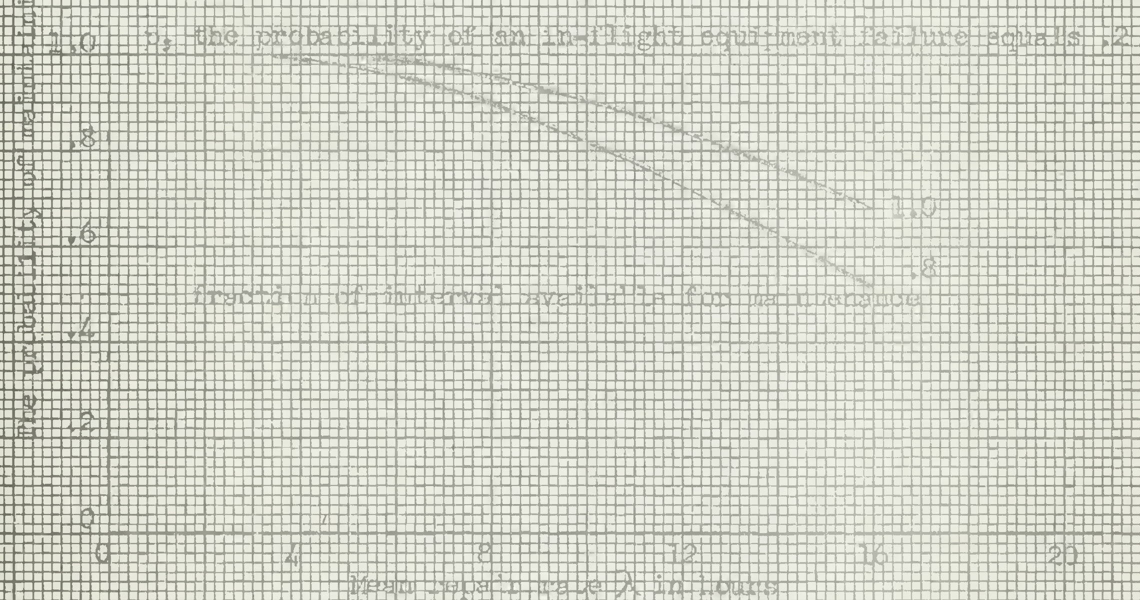


Figure 18